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# Resistivity at the Field Null of the FRC Plasma

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## I. SYNOPSIS

In the absence of major destructive instabilities, the configuration time is ultimately determined by particle and flux containment. If the profiles are "gentle," then the anomalous flux-loss rate depends essentially on the anomalous resistivity at the field null.

Conventional electrostatic quasi-linear models of anomalous cross-field resistive diffusivity are based upon the use of  $\vec{E} \times \vec{B}$  drift velocities, and hence break down at the magnetic field null. In this paper, an electromagnetic treatment valid at the field null is developed, based upon the presence of flute-parity perturbations. An expression for anomalous resistivity at the field null in the quasi-linear approximation is derived by averaging in the ignorable direction over the random phases of the perturbations. The expression is valid for arbitrary (non-local) radial shapes of the perturbing modes (for example, the eigenfunctions need not be centered at the field null), and for an arbitrary ratio of real frequency to growth rate. The effective resistivity due to flute perturbations of the MHD type will be considered.

## II. THE MODEL

Consider a quasi-1D model of the FRC. Revert to a slab model with "radial" direction  $x$ , "azimuthal" direction  $y$  (periodic), and axial direction  $z$ . The zero-order magnetic field is along the  $z$ -direction. We seek flute-type perturbations independent of  $z$  in a model in which the equilibrium has only an  $x$ -dependence. We neglect electron inertia, so Ohm's law reads

$$\vec{E} + \vec{u} \times \vec{B} = D_\eta \nabla \times \vec{B} - \frac{1}{ne} \nabla P_e, \quad (1)$$

where  $D_\eta = \eta/\mu_0$  is the resistive diffusivity of the plasma,  $\vec{E}$  and  $\vec{B}$  are electric and magnetic field vectors,  $P_e$  is electron pressure,  $n$  is electron number density, and  $u$  is the electron fluid velocity. (Thus, the Hall term is included.)

There are two types of modes independent of  $z$ . One has perturbation components  $\delta B_x$ ,  $\delta B_y$ , and  $\delta E_z$ . The other has perturbation components  $\delta E_x$ ,  $\delta E_y$ , and  $\delta B_z$ . It can be shown, by using potentials for  $\vec{E}$  and  $\vec{B}$  in Eq. (1), that modes of the former type always die away resistively as the perturbations are convected with the electron fluid. Consequently, we restrict attention to modes of the latter type. They have the form of interchanges.

## III. ELECTROSTATIC PERTURBATIONS

The conventional approach is to examine the effect of cross-field electrostatic perturbations,  $\delta E_y$ , on the cross field (radial) particle flux,  $nu_x$ . One takes the cross product of Eq. (1) with  $\vec{B}$ , multiplies by  $n$  and divides by  $B^2$ , and invokes radial pressure balance to find

$$nu = n \frac{\vec{E} \cdot \vec{B}}{B^2} = n \frac{n \nabla P_e \cdot \nabla P_e}{B^2} + \frac{n \nabla P_e \cdot \vec{B}}{c B^2}, \quad (2)$$

where  $P$  is the total plasma pressure. The second term on the rhs constitutes the classical contribution to the radial flux due to resistivity intrinsic to a quiescent plasma. It can be thought of as allowing a resistive slippage between the plasma velocity and the velocity of magnetic field lines,  $n(\vec{u} - \vec{V}_E)_x$ , where  $\vec{V}_E = \frac{\vec{E} \times \vec{B}}{B^2}$ . The third term has no zero-order radial (x) component because  $\nabla P_e$  itself is x-directed. Also, in a purely electrostatic model in which perturbations of  $\vec{B}$  are suppressed,  $(\nabla \delta P_e)_y$  perturbations (with  $\delta P_e$  periodic in y) will contribute no net effect to the radial particle flux when averaged over the y-direction.

The first term on the rhs can produce a nonlinear electrostatic contribution to the radial particle flux, of the form

$$(nu_x)_{NL} = \frac{\langle \delta n \delta E_y \rangle}{B_{z0}}, \quad (3)$$

in which perturbations of magnetic field are again suppressed in the electrostatic model. From this term, one could construct an equivalent anomalous resistivity from

$$\frac{\langle \delta n \delta E_y \rangle}{B_{z0}} = -D_{part}^* \frac{\partial n}{\partial x} \quad (4)$$

where the anomalous particle diffusivity is given by

$$D_{part}^* = (D_\eta^*) \left( \frac{1}{2} \beta \right) = r_{ce}^2 \nu_e^*, \quad (5)$$

wherein we have assumed uniform and equal temperatures for simplicity. Here,  $\beta = P/(B^2/2\mu_0)$ ,  $\nu_e^*$  is the anomalous electron collision frequency, and  $r_{ce}$  is the electron gyro-radius. Eq. (3) breaks down at or near the magnetic field null.

#### IV. ELECTROMAGNETIC PERTURBATIONS AT THE FIELD NULL

Write Eq. (1) as

$$ne\vec{E} + ne\vec{u} \times \vec{B} = \eta nc\vec{J} - \nabla P_e, \quad (6)$$

split into mean values depending only upon the x-coordinate and fluctuations depending upon the x-coordinate and also periodic in the y direction,

$$\begin{aligned} n &= n_0 + \delta n, & \vec{E} &= \vec{E}_0 + \delta \vec{E}, & \vec{u} &= \vec{u}_0 + \delta \vec{u} \\ \vec{B} &= \vec{B}_0 + \delta \vec{B}, & P_e &= P_{e0} + \delta P_e, \end{aligned} \quad (7)$$

take the y-component, average over the y direction,  $\langle \dots \rangle$ , and neglect the explicit resistive term (in order to concentrate on anomalous resistivity due to the fluctuations). Note that  $u_0 = u_{0y}(x)\hat{y}$ , and  $\partial P_{e0}/\partial y = 0$ . After dividing through by  $(n_0 e)$ , one finds

$$E_{0y} - \eta^* J_{0y} = \left\langle \frac{\delta n}{n_0} (\delta E_y - \delta u_x B_{0z}) \right\rangle + \langle \delta u_x \delta B_z \rangle = (\eta^* J_{0y})_1 + (\eta^* J_{0y})_2, \quad (8)$$

which defines the anomalous resistivity  $\eta^*$ . Henceforth, we shall refer to the two terms on the rhs of Eq. (8) as  $(\eta^* J_{0y})_1 + (\eta^* J_{0y})_2$ .

Now, the y-component of Eq. (1) for the fluctuations/reads

$$\delta E_y - \delta u_x B_{z0} = -\frac{\partial}{\partial y} \left( \frac{\delta P_e}{n_0 e} \right) \quad (9)$$

where we note that  $u_{x0} \equiv 0$  and  $\partial P_{e0}/\partial y = 0$ . Substitution of (9) into (3) yields, for the first term on the rhs,

$$(\eta^* J_{0y})_1 = \left\langle \frac{\delta n}{n_0^2 e} \frac{\partial}{\partial y} \delta P_e \right\rangle. \quad (10)$$

We next assume that the adiabatic law governs the electron fluid.

$$\left( \frac{\partial}{\partial t} + \vec{u} \cdot \nabla \right) (P_e \bar{\rho}^\gamma) = 0 \quad (11)$$

where the plasma mass density is  $\rho \equiv m_i n$  and  $\gamma$  is the adiabatic index. This equation linearizes to

$$\left( \frac{\partial}{\partial t} + u_{0y} \frac{\partial}{\partial y} \right) (\delta P_e - C_s^2 \delta \rho) + \delta u_x \left( \frac{\partial P_{e0}}{\partial x} - C_s^2 \frac{\partial \rho_0}{\partial x} \right) = 0 \quad (12)$$

where  $C_s^2 \equiv \gamma P_{e0}/\rho_0 \equiv \gamma T_{e0}/m_i$ . Note that  $\partial P_{e0}/\partial x$  and  $\partial \rho_0/\partial x$  vanish at the field null. Therefore, at the field null,

$$\delta P_e - C_s^2 \delta \rho = f(y - u_{0y} t). \quad (13)$$

Since  $f$  is periodic in  $y$ ,  $f$  must vanish identically for unstable perturbations. Therefore, at the field null,

$$\delta P_e = C_s^2 \delta \rho = \gamma T_{e0} \delta n \quad (14)$$

Substitution of (14) into (10) yields  $(\eta^* J_{0y})_1 = 0$  at the field null. Therefore, we turn to the second term on the rhs of Eq. (8), namely

$$(\eta^* J_{0y})_2 = \langle \delta u_x \delta B_z \rangle \quad (15)$$

To examine this term, we take the z-component of the curl of Eq. (1), linearized in the fluctuations, and evaluated at the field null. Noting that  $\nabla \cdot \vec{B} = 0$ ,  $\nabla \cdot \vec{u}_0 = 0$ ,  $\vec{B} \cdot \nabla \vec{u} = 0$  for flute perturbations, and that  $n_0$  and  $P_{e0}$  are independent of  $y$ , with  $\partial n_0/\partial x = 0$  and  $\partial P_{e0}/\partial x = 0$  at the field null, we find

$$\left( \frac{\partial}{\partial t} + u_{0y} \frac{\partial}{\partial y} \right) (\delta B_z) + \delta u_x \left( \frac{d}{dx} B_{z0} \right) = 0. \quad (16)$$

Thus, Eq. (16) determines  $\delta u_x$  in terms of  $\delta B_z$ , which, upon inserting into Eq. (15) yields, at the field null,

$$(\eta^* J_{0y})_2 = \left( \frac{dB_{z0}}{dx} \right)^{-1} \left\langle \delta B_z \left( \frac{\partial}{\partial t} + u_{0y} \frac{\partial}{\partial y} \right) \delta B_z \right\rangle. \quad (17)$$

The final term clearly vanishes upon averaging periodic perturbations over the  $y$  direction. Hence, Eq. (17) reduces to, at the field null,

$$\frac{\eta^*}{\mu_0} = \left( \frac{dB_{z0}}{dx} \right)^{-2} \left\langle \delta B_z \frac{\partial}{\partial t} \delta B_z \right\rangle. \quad (18)$$

In general, a growing mode, periodic in  $y$ , can be written as

$$\delta B_z = (ae^{i\psi} + a^*e^{-i\psi})e^{\Gamma t}; \quad (\psi = ky - \omega t), \quad (19)$$

where  $\Gamma$  is the growth rate and  $\omega$  is the real frequency of the mode. Upon use of this form in Eq. (18), the final result for  $\eta^*$  can be obtained by averaging over  $y$ . No time-average is required. The result is

$$\frac{\eta^*}{\mu_0} = \left( \frac{dB_{z0}}{dx} \right)^{-2} (2\Gamma|a|^2 e^{2\Gamma t}) = \left( \frac{dB_{z0}}{dx} \right)^{-2} (\Gamma|\delta B_z(t)|^2), \quad (20)$$

where the growing rms (with respect to  $y$ ) amplitude  $|\delta B_z(t)|$  must be assigned a saturated value from some nonlinear model or from experimental observations. Eq. (20) is the general quasilinear expression for the anomalous resistive diffusivity at the field null due to interchange-like modes. In general, one will sum such contributions over all active modes.

## V. ANOMALOUS RESISTIVITY AT THE FIELD NULL FROM INTERCHANGE-TYPE INSTABILITIES

For a growth rate that scales (for interchange or co-interchange ideal modes) as<sup>1</sup>

$$\Gamma \sim V_A / \left( \frac{1}{2} \ell_z \right) \quad (21)$$

where  $\ell_z$  is the length of a representative flux surface and  $V_A$  is a representative Alfvén speed on that surface, with  $(dB_{z0}/dx) \approx B_{ez}/(\frac{1}{2}r_s)$ , the anomalous resistive diffusivity at the field null becomes

$$\frac{\eta^*}{\mu_0} \sim \frac{1}{4} \left| \frac{\delta B}{B_{ez}} \right|^2 \frac{r_s V_A}{\epsilon}. \quad (22)$$

Here,  $\epsilon$  defines (non-standardly) the elongation ( $\frac{1}{2}\ell_z/r_s$ ) of the unstable flux surface, the field at the separatrix radius  $r_s$  is  $B_{ez}$ , and  $|\delta B|$  is the magnetic perturbation at the field null. This result suggests that larger radius and smaller elongation are detrimental to the effective resistivity at the field null. As a numerical example, we take  $B = 5 \times 10^3$  Gauss,  $n = 10^{15} \text{ cm}^{-3}$ ,  $r_s = 15 \text{ cm}$ ,  $\epsilon = 7$ ,  $V_A \approx 2 \times 10^7 \text{ cm/s}$  (deuterium),  $|\frac{\delta B}{B_{ez}}| \sim 0.1$ , and we then find  $(\eta^*/\mu_0) \sim 10^5 \text{ cm}^2/\text{s}$ , compared to a classical resistive diffusivity of  $\eta/\mu_0 \sim 10^4 \text{ cm}^2/\text{s}$  for  $T_e = 100 \text{ eV}$ .

1. D. C. Barnes, "Stability of Tilting Modes in Field Reversed Configurations," Document EUR 11335 EN, International School of Plasma Physics (Piero Caldirola) edited by S. Ortolani and E. Sindoni, Varenna, Italy (Sept. 1987).